Hardness of Network Satisfaction for Relation Algebras with Normal Representations

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THEORETICAL COMPUTER SCIENTISTS FOR FUTURE

Outline

- Definitions: Relation Algebras, Representations, Networks, ...

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- A class of computational problems: Network Satisfaction Problems
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- Taming RBCP: Normal Representations
- General hardness criterions for NSPs

Proper Relation Algebras

Definition

Let D be a set and $E \subseteq D^2$ an equivalence relation. Consider $(\mathcal{P}(E); \cup, \bar{}, 0, 1, \mathrm{Id}, \breve{}, \circ)$ with the following interpretation of function symbols:

1. $A \cup B := A \cup B$,
2. $\bar{A} := E \setminus A$,
3. $0 := \varnothing$,
4. $1 := E$,
5. $\mathrm{Id} := \{(x, x) \mid x \in D\}$,
6. $A^{\smile} := \{(x, y) \mid (y, x) \in A\}$,
7. $A \circ B := \{(x, z) \mid \exists y \in D : (x, y) \in A \text{ and } (y, z) \in B\}$.

A subalgebra of $(\mathcal{P}(E); \cup, \bar{}, 0, 1, \mathrm{Id}, \breve{}, \circ)$ is called proper relation algebra.

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For model theorists:

For a proper relation algebra $\mathcal{R}$ we view $\mathbb{R} = (D; \mathcal{R})$ as a relational structure.

Relation Algebras

Definition

A relation algebra $\mathcal{A}$ is an algebra $(A; \cup, \bar{}, 0, 1, \mathrm{Id}, \breve{}, \circ)$ of type $(2, 1, 0, 0, 0, 1, 2)$ satisfying the following laws:

1. $(A; \cup, \bar{}, 0, 1)$ is a boolean algebra,
2. $(x \circ y) \circ z = x \circ (y \circ z)$,
3. $(x \cup y) \circ z = x \circ z \cup y \circ z$,
4. $x \circ \mathrm{Id} = x$,
5. $(x^{\smile})^{\smile} = x$,
6. $(x \cup y)^{\smile} = x^{\smile} \cup y^{\smile}$,
7. $(x \circ y)^{\smile} = y^{\smile} \circ x^{\smile}$
8. $(x^{\smile} \circ \overline{(x \circ y)}) \cup \bar{y} = \bar{y}$.

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Point Algebra:

The set $\{=, <, >, \leq, \geq, \varnothing, \neq, \mathbb{Q}^2\}$ together with the "natural" relation algebra operations and the table on the right.

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Representations

Definition

Let $\mathbf{A}$ be a relation algebra. A relational structure $\mathfrak{B}$ is a representation of $\mathbf{A}$ if

- $\mathfrak{B}$ is an A-structure,
- the induced proper relation algebra on a subset of $\mathcal{P}(B^2)$ is isomorphic to $\mathbf{A}$.

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Examples

- $(\mathbb{Q}; =, <, >, \leq, \geq, \varnothing, \neq, \mathbb{Q}^2)$ is a representation of the Point Algebra.
- The countable, universal, homogeneous, triangle-free graph

$$\mathbb{H} = (V; =, E, N, E \cup =, E \cup N, N \cup =, \varnothing, V^2)$$

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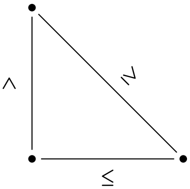

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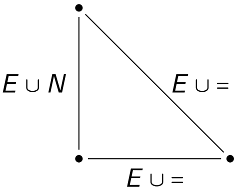

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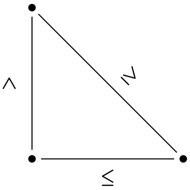

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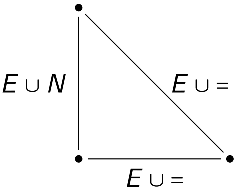

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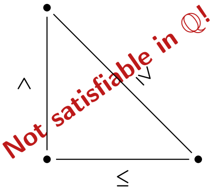

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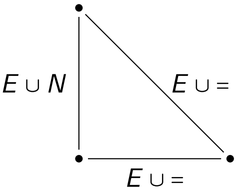

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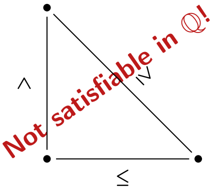

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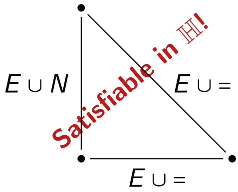

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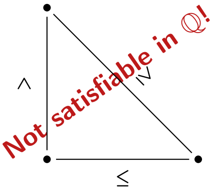

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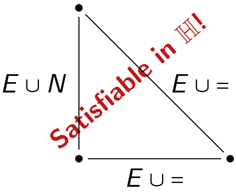

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- There exists a finite relation algebra with an undecidable NSP (Hirsch 1999).
- Focus on subclass of finite relation algebras with "nice" representations.

Normal Representations

Definition

Let $\mathbf{A}$ be a relation algebra. An $\mathbf{A}$-network $(V; f)$ is called atomic if the image of f only contains atoms and if

$$f(a, c) \leq f(a, b) \circ f(b, c)$$

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- homogeneous if every isomorphism of finite substructures of $\mathfrak{B}$ can be extended to an automorphism;
- normal if it is fully universal, square and homogeneous.

Result 1

Let **A** be a finite relation algebra with a normal representation $\mathfrak{B}$.
Suppose there exists $e \in A$ such that $e^{\mathfrak{B}}$ is a non-trivial equivalence relation with finitely many classes. Then NSP(**A**) is NP-complete.

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Application:

Consider RA #13[1] with domain $\mathcal{P}(\{=, E, N\})$,
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Let **A** be a finite relation algebra with a normal representation $\mathfrak{B}$. Suppose that for all $e \in A$ the relation $e^{\mathfrak{B}}$ is not a non-trivial equivalence relation and let a be a symmetric atom of **A** such that $a \nleq a \circ a$ holds. Then NSP(**A**) is NP-complete.

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N	N	$\{E, N\}$	$\{=, E, N\}$

- The countable, universal, homogeneous, triangle-free graph is a normal representation of RA #17.
- No element of $\mathcal{P}(\{=, E, N\})$ induces a non-trivial equivalence relation.
- $E \nleq E \circ E$ holds.
- NSP(#17) is NP-complete.

[1] Enumeration due to Andréka and Maddux: "Representations for small relation algebras".

Open Problems

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- relation algebras where all non-trivial equivalence relations have infinitely many infinite classes.
- relation algebras with only symmetric atoms.

Thank you for your attention!